### Lab07 – Complex arithmetics

# Content

2. Learning objectives

3. Exercises

3.1 Basic Exercises

3.2 Bridging exercises

3.2.1 Solving quadratic equations with complex solutions

3.2.2 Euler notation of complex numbers

# Learning objectives

## Exam objectives

By the end of this lab you should be able to (pen and paper):

* Perform complex arithmetics: addition, subtraction, multiplication, division and exponentiation
* Simplify complex (alfa)numerical expressions
* Convert between both the algebraic and trigonometric shape of a complex number
* Draw complex number in their Gaussian plane

We advise you to **make your own summary of topics** which are new to you.

## Supportive objectives

### Self-support by GeoGebra

More specifically related to the above you should in GeoGebra:

* Compute complex arithmetics: addition, subtraction, multiplication, division and exponentiation
* Visualize complex numbers
* Convert between both the algebraic and trigonometric shape of a complex number

# Exercises

Dependent of the lab session you may work individually or teamed (organized by the lab attendant). In either case make sure that throughout the course of this lab, you backup sufficiently your solution file on your local machine as

**1DAExx-0y-name1**(+name2+name3).GGB given **xx**=groupcode, **0y**=labindex

Use <https://www.geogebra.org/classic>

## Basic exercises

For all underneath exercises do at least one pen and paper attempt in your own workbook, before checking your results in GeoGebra. In case of mismatches between your handwritten results and GeoGebra’s output, do not hesitate to seek assistance by your Lab attendant, as you are owner of your own learning.

### Calculate the modulus and the argument of the following complex numbers

1. 3 + i.4 |z1| = sqrt(32 + 0.42) = 3.03 θ = atan2(0.4, 3) = 7.59 degrees
2. -i + 1
3. i
4. 2

### Write the following complex numbers in trigonometric or polar notation. ( a+ i.b -> |r|(cos α + i . sin α )

1. 3 – i.4 z1 = |z1| ( cosθ + isinθ) → 3.03 (cos 7.59 + isin 7.59)
2. 1 + i
3. 0.72 + i . 0.72
4. -i
5. 1

### Evaluate the following complex expressions and write the result in algebraic/cartesian form (a+bi)

1. (2 + i . 3) I -3 + 2i
2. (-1 + i . 2)(-2 + i . 3) -4 -7i
3. (2 – i)(-1 + i)
4. (2.i -2)(-i + 2)
5. (2 – i ) / i
6. (1 - i.3)/(2 + i.2)
7. 1 / (2 – i)
8. i / (1 + i)

## Bridging exercises

### Solving quadratic equations with complex solutions

For any quadratic equation : the roots can be found using :

Applying the fact that or gives us the means to solve quadratic equations with complex solutions.

Solve the following quadratic equations using the above mentioned information.

### Euler notation of complex numbers

Given a complex number z = a + i . b we know that the polar or trigonometric notation of z =r (cos α + i . sin α ). We can also write c in the Euler notation which gives

Where

And

Exercise : Watch and learn

1. Visualize the following complex numbers and in Geogebra
2. Write them in Euler notation
3. Multiply and
4. Visualize the result in Geogebra
5. Observe and write down what happened
6. ,
7. ,
8. ,
9. ,

What can you conclude ? Try to formulate your conclusions. Next weeks lab will continue on this.

## Contextual practice

### Power of a complex number

When calculating a fractal pattern or a rotation, the same operation is repeatedly applied to a complex number. We will look at the concept of taking the power of a complex number to develop an intuition:

C1 = 0.833 + *i* /2

Calculate following powers and the modulus and argument of the resulting complex number:

|  |  |  |  |
| --- | --- | --- | --- |
| **Power** | **Complex number** | **Modulus** | **Argument (degrees)** |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |

What do you conclude? Why do you get this result?

Formulate a proof that the power **n** of a complex number with modulus equals to 1 will always result in a complex number with modulus one.

What can you say about the argument when the power is **n** ?

(Skip ahead to the last page for a tip, but recommended to think about it first)

**Tips**:

For the proof of the power function, the euler notation as written in section 3.2.2 is a fast way to develop a proof in one line.